International Journal of Theoretical Physics, Vol. 22, No. 6, 1983

Analysis of the Einstein–Podolsky–Rosen Experiment by Relativistic Quantum Logic

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Received September 16, 1981

The EPR experiment is investigated within the abstract language of relativistic quantum physics (relativistic quantum logic). First we show that the principles of reality (R) and locality (L) contradict the validity principle (Q) of quantum physics. A reformulation of this argument is then given in terms of relativistic quantum logic which is based on the principles R and Q. It is shown that the principle L must be replaced by a convenient relaxation $\tilde{L}$, by which the contradiction can be eliminated. On the other hand this weak locality principle $\tilde{L}$ does not contradict Einstein causality and is thus in accordance with special relativity.

1. INTRODUCTION

The experiment of Einstein, Podolsky, and Rosen (hereafter referred to as EPR) describes a quantum physical system consisting of two particles which, according to quantum mechanics, are correlated with respect to the values of their observables although they cannot interact any longer and are separated by a spacelike distance. Einstein, Podolsky, and Rosen (1935) advanced this *Gedankenexperiment* originally in order to show that the nonlocal quantum mechanical correlations cannot correspond to correlations between the real situations of the two particles and occur only because of the incompleteness of quantum mechanics. This result initiated a series of investigations concerning the question whether quantum mechanics can be completed by means of hidden variables or not. Finally, Bell (1964) showed that the EPR experiment can be described by hidden variables only if they possess a nonlocal behavior with respect to the two subsystems also. This conclusion lead many physicists to accept that there exist some kind of nonlocal correlations between the real situations of the two subsystems.

However, the nature of the quantum physical nonlocality is often obscured in the literature and confused with the possibility of superluminal signals which would violate the Einstein causality.

It is the purpose of this paper to clarify the conclusion which has to be drawn from Bell's result. Since hidden variables did not turn out to be significant for the EPR experiment, because they inherit a nonlocal behavior, we refer to the quantum mechanical description of the *Gedankenexperiment*. Although Bell's investigation concerns a hidden-variable description, it is possible to dispense with hidden variables and transfer Bell's result to a restriction concerning the interpretation of quantum mechanics. We presuppose that the interpretation is realistic in the sense that it makes use of a language about individual quantum mechanical systems and their properties. On the other hand, the language is very general and appropriate for any physical system. The formal structure of this language is also called "quantum logic" by the authors. In Section 2 of the paper it is shown that the principles underlying the argumentation of EPR, the *reality* and *locality principles*, which are consistent with our presupposition, lead to special relations among the propositions of the language. These relations can be used to derive an inequality between probability expressions with respect to propositions, which is the analog to Bell's inequality within the framework of the hidden-variables description. It is well known that Bell's inequality contradicts the statistical predictions of quantum mechanics which, nowadays, are strongly confirmed by realizations of the EPR experiment. Hence, the EPR principles contradict the validity of quantum mechanics within the very general framework of the interpretation applied in this paper.

In order to resolve the contradiction, the EPR principles must be altered in such a way that they do not allow the derivation of the above-mentioned special relations among propositions (which give rise to Bell's inequality). For the further investigation of the principles, we at first give a short account of the general quantum language which has been developed in previous papers and, then, apply it to the EPR experiment. This is done in Section 3 of the paper. It is taken into account that the quantum language should allow a description of quantum physical systems in space-time. This leads to a more concrete language where the validity of propositions depends on some validity regions in space-time. Since the relativistic limitations of signals are taken into account for the communication of proof results of propositions between different observers, the resulting language is called "relativistic quantum language." Indeed, the quantum physical restrictions of this language which come from the incommensurability of propositions, and the relativistic restrictions, which come from light-cone structure of space-time, are most apparent in the case of the EPR experiment.

Within the framework of the relativistic quantum language the EPR principles are reconsidered. It is shown in Section 4 that the reality principle is already incorporated into the language as a criterion of truth of propositions. This principle is a precondition of a *real* description of quantum physical systems and is not questioned here. On the other hand, the locality principle is not consistent with the description of a system by means of the relativistic quantum language and must be altered. We formulate a weak locality principle which is consistent with our language, i.e., which still respects the relativistic limitations of signals due to the light-cone structure of space-time and which does not contradict the validity of quantum mechanics via Bell's inequality. Hence the principle of *Einstein causality* need not be altered in order to account for the nonlocal correlations between the real situations of the two subsystems in the EPR experiment.

2. THE EPR CONTRADICTION

The analysis of the principles underlying the argumentation of Einstein, Podolsky, and Rosen (1935), leads to the result that the principles which allow for deriving the incompleteness of quantum physics are contradictory. The demonstration of this contradiction goes back to a result by Bell (1964) and will be reconstructed here in a simpler version due to Wigner (1970). In order to derive the contradiction, to discuss the physical significance of the principles, and finally to resolve the contradiction by changing one of them, we shall at first reformulate the principles of Einstein, Podolsky, and Rosen in a more rigorous and systematic way.

2.1. The EPR Principles. The principles are the *reality principle* (R), which is formulated by EPR as a sufficient condition for an element of reality (a property as we say) pertaining to a physical system, the *locality principle* (L), which is implicitly assumed by EPR but is an important principle within their argumentation, and the *validity principle* (Q) of quantum theory concerning the *Gedankenexperiment*. Our reformulation of these principles is given by the following scheme:

$R :=$	The value of a physical magnitude A can be determined without in any way altering the physical system S.	$\frown$	A property $\mathcal{P}$ which corresponds to the value of the magnitude A pertains to the system S.
	$\Vert$ R_1		$\Vert$ R_2

$L:=$	Two systems cannot interact with each other.	$\curvearrowright$	A measurement with respect to one system cannot alter the other system in any way.
	$\parallel$ $\ddot{L}_1$		$\parallel$ $\ddot{L}_2$

$Q:=$	The von Neumann–Dirac quantum theory is valid

where $\curvearrowright$ denotes the metalinguistic implication "if... then... ."

These principles will be applied with respect to a *Gedankenexperiment* of the following kind:

We consider a pair $S = S_{\mathrm{I}} + S_{\mathrm{II}}$ of two systems S_{I} and S_{II} such that S_{I} and S_{II} are strongly correlated with respect to three pairwise incommensurable magnitudes A, B, and C. Let us assume for simplicity that these magnitudes have only two possible values designated by "yes" and "no." *Strong correlation* means here that if the magnitude A, say, has been measured with respect to one system and the result is "yes," then the value of the magnitude A with respect to the other system is "no" and vice versa. Two magnitudes are *incommensurable* if and only if the result of the measurement of one magnitude is altered by the measurement of the other magnitude. We assume throughout the paper that measurements are *ideal measurements of the first kind* which do not alter the system more than necessary in order to determine the value of the corresponding magnitude.

As an example of this *Gedankenexperiment* we refer to a pair of two spin-1/2 particles in an $^1S_0$ state, e.g., two protons in the singlet state after a proton–proton interaction. This example goes back to Bohm (1951). With respect to each of the two systems S_{I} and S_{II} the spin observable $\sigma(\vartheta)$ in a certain direction **a** with $a_z = \cos\vartheta$ can be measured. We denote the spin observables with respect to the systems S_{I} and S_{II} by $\sigma_{\mathrm{I}}(\vartheta)$ and $\sigma_{\mathrm{II}}(\vartheta)$, respectively. Two spin observables with respect to one system, $\sigma_{\mathrm{I}}(\vartheta)$ and $\sigma_{\mathrm{I}}(\vartheta')$, say, are incommensurable if $\vartheta' \neq \vartheta \bmod \pi$. We choose $A \mathrel{\hat{=}} \sigma(\vartheta)$, $B \mathrel{\hat{=}} \sigma(\vartheta')$, and $C \mathrel{\hat{=}} \sigma(\vartheta'')$ for appropriate ϑ, ϑ', and ϑ'' such that A, B, and C are pairwise incommensurable. The operator $\sigma_{\mathrm{I}}(\vartheta)$ is defined on the Hilbert space $\mathcal{H}_{\mathrm{I}}$ with respect to system S_{I}; its eigenstates are $\varphi^{\mathrm{I}}_{+}(\vartheta)$ and $\varphi^{\mathrm{I}}_{-}(\vartheta) \in \mathcal{H}_{\mathrm{I}}$ with the corresponding eigenvalues $+1$ and -1. The fact that the observable $\sigma_{\mathrm{I}}(\vartheta)$ has the value $+1$ [i.e., that the system S_{I} is in the state $\varphi^{\mathrm{I}}_{+}(\vartheta)$ and therefore $\sigma_{\mathrm{I}}(\vartheta)\varphi^{\mathrm{I}}_{+}(\vartheta) = +1 \cdot \varphi^{\mathrm{I}}_{+}(\vartheta)$] corresponds to the property $\mathcal{P}_{a_{\mathrm{I}}}$ and the fact that the observable $\sigma_{\mathrm{I}}(\vartheta)$ has the value -1 corresponds to the property $\mathcal{P}_{\neg a_\mathrm{I}}$. For the comparison with the above principles it is convenient to use the propositions a_I:="The property $\mathcal{P}_{a_\mathrm{I}}$ pertains" and $\neg a_\mathrm{I}$:="The property $\mathcal{P}_{\neg a_\mathrm{I}}$ pertains." With respect to the values of the observables A, B, C we have the propositions $a_I, \neg a_\mathrm{I}, b_\mathrm{I}, \neg b_\mathrm{I}, c_\mathrm{I}, \neg c_\mathrm{I}$ or, more precisely for our example $a_\mathrm{I}(\vartheta_+)$, $a_\mathrm{I}(\vartheta_-)$, $b_\mathrm{I}(\vartheta_+)$, $b_\mathrm{I}(\vartheta_-)$, $c_\mathrm{I}(\vartheta_+), c_\mathrm{I}(\vartheta_-)$. For system S_II the notions are analogous. The ${}^\mathrm{I}S_0$ state of $S = S_\mathrm{I} + S_\mathrm{II}$ is given by the vector $\phi := (1/\sqrt{2})[\varphi_+^\mathrm{I}(\vartheta)\otimes\varphi_-^\mathrm{II}(\vartheta) - \varphi_-^\mathrm{I}(\vartheta)\otimes\varphi_+^\mathrm{II}(\vartheta)]$ in the Hilbert space $\mathcal{H} = \mathcal{H}_\mathrm{I}\otimes\mathcal{H}_\mathrm{II}$.

After the preparation of the pair of strongly correlated systems S_I and S_II, the two systems are separated into two space regions (i.e., regions with high registration probabilities for S_I an S_II, respectively), and there is no interaction between the systems. If now the magnitude A is measured with respect to system S_I and the result is "yes," we know because of the correlation that the value of A with respect to system S_II is "no." Since the premise of the locality principle L is satisfied, the conclusion L_2 is valid, which means that system S_II is not altered in any way. Hence the premise of the reality principle R is also satisfied where the value of A is even determined, and we conclude that the property $\mathcal{P}_{\neg a}$ pertains to system S_II after the preparation. The argumentation is summarized by Figure 1.

Since because of L the premise of R is satisfied for any of the magnitudes A, B, and C, i.e., since the values of the magnitudes A, B, and C with respect to system S_II can be determined without altering system S_II in any way, all properties which correspond to the values of the magnitudes pertain to system S_II after the preparation. In this case we say that the magnitudes of system S_II are simultaneously *objectified*. Under these conditions we have:

$$(a_\mathrm{II} \cup \neg a_\mathrm{II}) \cap (b_\mathrm{II} \cup \neg b_\mathrm{II}) \cap (c_\mathrm{II} \cup \neg c_\mathrm{II}) \equiv V \tag{1}$$

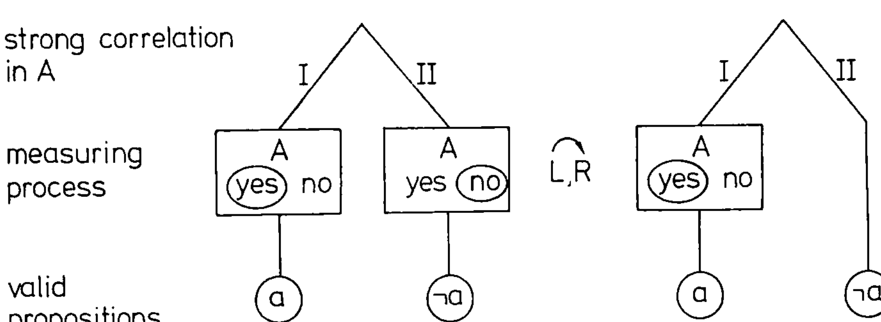


Fig. 1. The objectivity of $\mathcal{P}_{\neg a}$ after the preparation of S_II.

where V denotes the "always true" proposition and $\cap$ and $\cup$ denote the logical connectives "and" and "or," respectively (which are systematically introduced in Section 3.1).

Since the same argumentation can also be applied to system S_{I} we have

$$(a_{\mathrm{I}} \cup \neg a_{\mathrm{I}}) \cap (b_{\mathrm{I}} \cup \neg b_{\mathrm{I}}) \cap (c_{\mathrm{I}} \cup \neg c_{\mathrm{I}}) \equiv V \tag{2}$$

also. Because of the pairwise incommensurability of the magnitudes A, B, and C, the value of at most one of the magnitudes can actually be measured with respect to one system and, by means of the correlations, the values of at most two of the magnitudes can actually be determined with respect to one system. Although the value of the other magnitude is objectively decided with respect to each system and respects the correlation, it is subjectively unknown. Only probabilities can be established for this magnitude within series of measurements with respect to a large set of pairs of correlated systems.

We show that a further elaboration of the above *Gedankenexperiment* leads to a result, a simplified version of Bell's inequality, which contradicts quantum theory in the above-mentioned example:

$$L\&R\&Q \cap \bar{\Lambda}$$

where $\bar{\Lambda}$ denotes the metalinguistic "always false" proposition.

In order to derive the contradiction we consider three series of measurements, each with respect to a large number of pairs of correlated systems. Each series consists in the measurements of two different magnitudes with respect to the two systems S_{I} and S_{II} of a pair. In the first series the magnitude A is measured with respect to system S_{I} and the magnitude B is measured with respect to system S_{II}; in the second series the magnitudes A and C and in the third series the magnitudes C and B are measured in the same way. By means of the relative frequencies of the "yes" results in both of the cases, the probabilities for the occurrence of these results can be experimentally determined. For a particular example of the *Gedankenexperiment* these probabilities can also be determined by quantum theory. The validity principle says that the quantum theoretical probabilities coincide with the experimentally detected probabilities. Let $p(a_{\mathrm{I}}, b_{\mathrm{II}})$, $p(a_{\mathrm{I}}, c_{\mathrm{II}})$, and $p(c_{\mathrm{I}}, b_{\mathrm{II}})$ denote these probabilities with respect to the first, second, and third series of measurements, respectively.

A relation between these probabilities is predicted by the following conclusion, which involves the locality and the reality principles as they are used in the above argumentation. Each pair of measurements with respect to the two systems S_{I} and S_{II} determines the values of the corresponding magnitudes for each system and, therefore, decides on the truth and falsity of the corresponding propositions. The "yes" results in the first series of measurements establish the truth of $a_I \cap \neg b_I$ and $\neg a_{II} \cap b_{II}$ with respect to S_I and S_{II}, respectively. It is sufficient to consider the propositions with respect to S_I since the propositions with respect to S_{II} are given by means of the correlations. From the above relation (1) it follows because of $a_I \cap \neg b_I \equiv a_I \cap \neg b_I \cap V$ that $a_I \cap \neg b_I \equiv a_I \cap \neg b_I \cap (c_I \cup \neg c_I)$. Thus we have for the probability $p_I(a \cap \neg b)$ that a and $\neg b$ are true with respect to system S_I:

$$p_I(a \cap \neg b) = p_I(a \cap \neg b \cap c) + p_I(a \cap \neg b \cap \neg c)$$

Analogously we have

$$p_I(a \cap \neg c) = p_I(a \cap \neg c \cap b) + p_I(a \cap \neg c \cap \neg b)$$

$$p_I(c \cap \neg b) = p_I(c \cap \neg b \cap a) + p_I(c \cap \neg b \cap \neg a)$$

It follows from these three equations that

$$p_I(a \cap \neg b) \leqslant p_I(a \cap \neg c) + p_I(c \cap \neg b)$$

Since the "yes" results of the measurements of A and B with respect to the two systems S_I and S_{II} actually determine the truth of the proposition $a_I \cap \neg b_I$, it follows that the probability $p(a_I, b_{II})$ must be equal to the probability $p_I(a \cap \neg b)$. Therefore, the above inequality must also hold with respect to the experimentally or theoretically established probabilities for the "yes" results of the pairs of measurements. In this way we arrive at the prediction of the above inequality for these probabilities:

$$p(a_I, b_{II}) \leqslant p(a_I, c_{II}) + p(c_I, b_{II})$$

which is a simplified version of Bell's inequality for the *Gedankenexperiment* considered here. The derivation of this inequality is summarized by Figure 2.

In our above example of the pair of two spin-1/2 particles in the $^1S_0$ state, the quantum theoretical probabilities are

$$p_\phi(a_I, b_{II}) = \tfrac{1}{2}\sin^2\tfrac{1}{2}\theta_{a,b}$$

$$p_\phi(a_I, c_{II}) = \tfrac{1}{2}\sin^2\tfrac{1}{2}\theta_{a,c}$$

$$p_\phi(c_I, b_{II}) = \tfrac{1}{2}\sin^2\tfrac{1}{2}\theta_{c,b}$$

where $\theta_{a,b}$ is the angle between the directions **a** and **b** corresponding to the

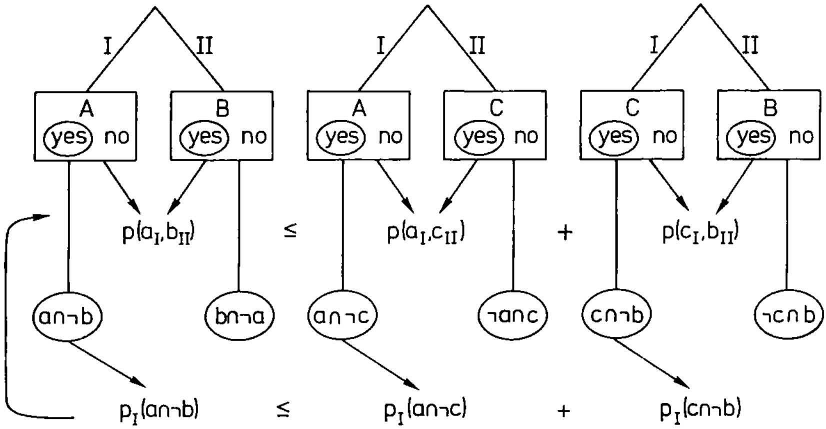


Fig. 2. The derivation of the inequality.

observables $\sigma(\vartheta)$ and $\sigma(\vartheta')$, etc. But the predicted inequality

$$\sin^2\tfrac{1}{2}\theta_{a,b} \leqslant \sin^2\tfrac{1}{2}\theta_{a,c} + \sin^2\tfrac{1}{2}\theta_{c,b}$$

cannot be satisfied for arbitrary angles. For instance it is easy to demonstrate that the inequality is not valid if **a**, **b**, and **c** are coplanar. Thus we have finally derived the contradiction

$$L \& R \& Q \cap \Lambda$$

which, indeed, can be called an EPR paradox in the strong sense of a contradiction.

2.2. The Evaluation of the Principles. In order to resolve the contradiction between the three principles L, R, and Q at least one of them has to be altered.

Concerning Q, the question has been investigated whether quantum theory is valid in the special case of correlated systems or not. A series of experiments has been performed as realizations of the *Gedankenexperiment* and has confirmed Q (Wu and Shaknov, 1950; Kocher and Commins, 1967; Kasday, 1971; Kasday et al. 1970; Freedman and Clauser, 1972; Clauser, 1976; Lamehi-Rachti and Mittig, 1976; Fry and Thompson, 1976; Aspect, Grangier, and Roger, 1981. For a review compare Clauser and Shimony, 1978).

The reality principle R is a meaningful principle concerning the conception of the language of quantum physics. It seems to establish a minimal connection between the observational language of physics and the concept of properties of a physical system. This means that whenever a language for physics is constituted as a language about physical systems and their properties, the reality principle R must be accepted.

The retention of the two principles R and Q leads to the query of the locality principle L. As the above argumentation shows, the EPR contradiction can only be resolved if it is assumed that the measurement of a magnitude with respect to one system alters the other system in some way. In this case, and only in this case, the reality principle can no longer be applied to deduce the validity of the above equations (1) and (2) and the simultaneous validity of two propositions with respect to each system in the above experiment. Indeed, the necessity of the locality principle L is not demanded by any physical considerations. In particular, it does not follow from the theory of relativity. The theory of relativity incorporates only the weaker principle K of *Einstein causality*, which can be formulated for our aims as

$K :=$	Two systems cannot interact with each other.	$\frown$	A measurement of a property with respect to S_I cannot alter S_{II} in such a way that a transmission of signals from S_I to S_{II} with $v > c$ becomes possible.
	$\Vert$ .. L_1		$\Vert$ .. K_2

The principle K should still be valid in a relativistic quantum theory. It follows from these considerations that the locality principle L has to be altered into a weaker principle $\tilde{L}$ such that the following holds:

(i) $L \frown \tilde{L}$

(ii) $\neg(\tilde{L} \frown \neg K)$ (i.e., the negation of K is not derivable from $\tilde{L}$)

(iii) $\neg(\tilde{L} \& R \& Q \frown \Lambda)$ (i.e., the principle $\tilde{L}$ must not contradict R and Q).

The question how to replace L by the weaker principle $\tilde{L}$ which is reasonable for quantum physics and satisfies the conditions (i) to (iii) can only be answered within the framework of relativistic quantum physics. A general language for relativistic quantum physics will be applied in Section 3 and the weaker locality principle $\tilde{L}$ will finally be formulated in Section 4.

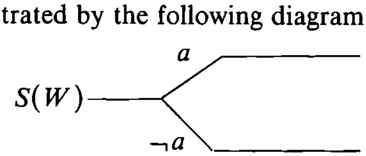
3. THE DESCRIPTION WITHIN THE GENERAL LANGUAGE OF RELATIVISTIC QUANTUM PHYSICS

In order to compare the space-time description of the system $S = S_{\mathrm{I}} + S_{\mathrm{II}}$ with the EPR principles, we apply the abstract language of relativistic quantum physics to S. This formal language $\mathbb{S}_U$ is built up from elementary propositions which are interpreted as statements about certain properties with respect to S. The EPR principles can then be regarded as metalinguistic statements about $\mathbb{S}_U$, the validity of which will be examined in Section 4.

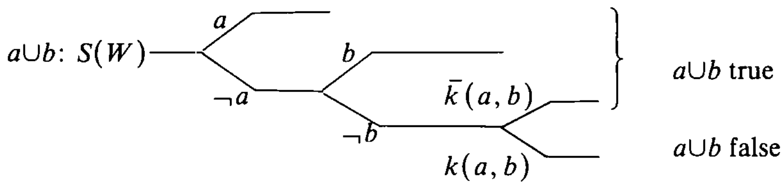
For the construction of the language $\mathbb{S}_U$, also called the *universal language* for S, two different aspects must be taken into account. At first we consider the language $\mathbb{S}(\mathfrak{R})$ of an observer who is located in a well-defined space-time region $\mathfrak{R}$ of the Minkowski space $\mathfrak{M}$ and who performs his experiments in $\mathfrak{R}$. $\mathbb{S}(\mathfrak{R})$ is also called the *local language* with respect to $\mathfrak{R}$. $\mathbb{S}(\mathfrak{R})$ is the formal quantum language without any relativistic constraints. A detailed investigation of the formal language has been given by Stachow (1976, 1978), Mittelstaedt (1978), Mittelstaedt and Stachow (1978) and Stachow (1980, 1981a,b). Since it is assumed in our *Gedankenexperiment* that spin measurements are performed with respect to the systems S_{I} and S_{II} in local regions $\mathfrak{R}_{\mathrm{I}}$ and $\mathfrak{R}_{\mathrm{II}}$, respectively, we shall consider the local languages $\mathbb{S}(\mathfrak{R}_{\mathrm{I}})$ and $\mathbb{S}(\mathfrak{R}_{\mathrm{II}})$, which are the formal quantum languages for the two systems S_{I} and S_{II}. In a next step, which is justified in Section 3.2, we shall consider the language $\mathbb{S}_U$ for the compound system $S = S_{\mathrm{I}} + S_{\mathrm{II}}$. Since this universal language comprises the local languages $\mathbb{S}(\mathfrak{R}_{\mathrm{I}})$ and $\mathbb{S}(\mathfrak{R}_{\mathrm{II}})$, we must take into account the relativistic constraints with respect to its propositions.

3.1. The Local Language. The formal language for one of the systems, S_{I}, say, can easily be constructed. For a general account of the formal quantum language we refer to Stachow (1980). However, before we shall apply it to the special case of a spin-1/2 system, it is useful to summarize its main features.

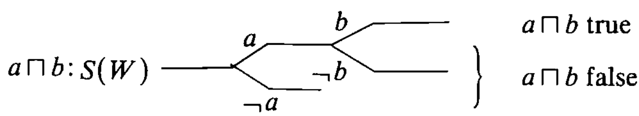
The language is constituted as a language for an individual physical system S and its properties. Those propositions, the truth and falsity of which can be decided by ideal measurements of the first kind, are called *material propositions*. If a material proposition a is tested with respect to a system S the preparation of which is formulated by the proposition W and if a is true, we write $S(W) \vdash a$. If a is false we write $S(W) \vdash \neg a$. The proof possibilities are illustrated by the following diagram also:



Compound propositions are generated by the material propositions and the *logical connectives* $\cap$, $\cup$, $\supset$, $\sim$ and the *sequential conjunction* $\sqcap$ which can be defined by proof trees. For example,



where the branching represents the temporal order of successive experimental tests, $k(a, b)$ states the commensurability, and $\bar{k}(a, b)$ states the incommensurability of a and b;



Two propositions A and B are said to be *value equivalent* ($A = B$) if and only if one proposition can be replaced by the other one without thereby influencing the value of the proposition. A and B are said to be *proof equivalent* ($A \equiv B$) if and only if within any proof process in which one of the two propositions occurs it can be replaced by the other one without thereby influencing the outcome of the proof process. Obviously the latter equivalence is stronger than the first one. It is convenient to proceed from propositions to equivalence classes with respect to the proof equivalence $\equiv$.

We consider the experimentally well-confirmed hypothesis that to each logically connected proposition A there corresponds a material proposition a such that A and a are value equivalent. This hypothesis means that a proof of A, instead of applying the procedure according to the proof tree for A, can also be performed by a measuring process for a without thereby influencing the values of the propositions. This hypothesis imposes a structure on the set E of material propositions mod $\equiv$ which is an *orthomodular lattice* $\langle E, \wedge, \vee, \neg \rangle$ such that the lattice elements are inductively given by $A \wedge B = A \cap B$, $A \vee B = A \cup B, \neg A = \sim A$.

The set of logically connected propositions mod $\equiv$ forms a *quasi-Boolean algebra* (cf. Rasiowa, 1974) and the set of sequential conjunctions a *Baer*-semigroup* (cf. Stachow, 1980).

In the special case of a spin-1/2 system S, the set of material propositions $E(S)$ is given by the set $\{a(\vartheta_+)\}_\vartheta \cup \{\Lambda, V\}$, where ϑ runs through all angles from $0°$ to $180°$, Λ is the "always false," and V the "always true" proposition the proofs of which do not afford any experimental manipulation with respect to the system. The lattice $\langle E(S), \wedge, \vee, \neg \rangle$ is illustrated by

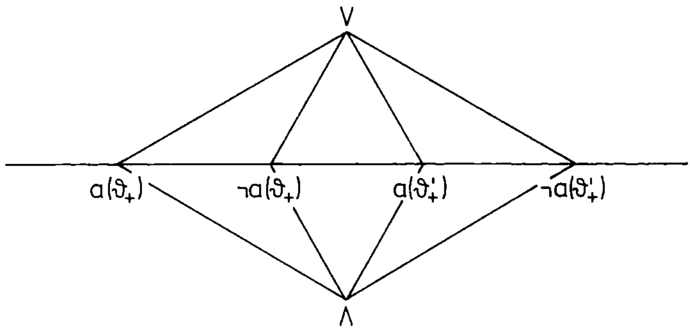


Fig. 3. The lattice of a spin-1/2 system.

Figure 3, where the horizontal line represents the continuum of all propositions $a(\vartheta_+)$ with $\neg a(\vartheta_+) := a(\vartheta_-) := a(180° - \vartheta)$, and the other lines represent the *implication relation* $\leqslant$, which is given by $A \leqslant B \cap A \equiv A \wedge B$.

The above language refers to an observer who is located within a finite, compact space-time region $\mathfrak{R} \subseteq \mathfrak{M}$ and performs the proofs of the propositions in $\mathfrak{R}$. In order to describe the EPR *Gedankenexperiment* we now consider two local languages $\mathfrak{S}(\mathfrak{R}_{\mathrm{I}})$ and $\mathfrak{S}(\mathfrak{R}_{\mathrm{II}})$ which refer to two space-like separated regions $\mathfrak{R}_{\mathrm{I}}, \mathfrak{R}_{\mathrm{II}} \subseteq \mathfrak{M}$ where spin measurements are performed with respect to the two spin-1/2 systems S_{I} and S_{II}. The discussion in Section 3.2 of this section will show that the universal language for the description of the compound system $S = S_{\mathrm{I}} + S_{\mathrm{II}}$ is generated by the two local languages $\mathfrak{S}(\mathfrak{R}_{\mathrm{I}})$ and $\mathfrak{S}(\mathfrak{R}_{\mathrm{II}})$ under the condition that $k(a_{\mathrm{I}}(\vartheta), a_{\mathrm{II}}(\vartheta')) \equiv V$ for all ϑ, ϑ', i.e., all material propositions with respect to system S_{I} are commensurable with all material propositions with respect to system S_{II}. The lattice $\langle E(S), \wedge, \vee, \neg \rangle$ which is generated by the two lattices $\langle E(S_{\mathrm{I}}), \wedge, \vee, \neg \rangle$ and $\langle E(S_{\mathrm{II}}), \wedge, \vee, \neg \rangle$ under this condition is very complicated and has not yet been investigated in detail. However, if the completeness, the atomicity, and the covering law are added as further axioms, its Hilbert space representation leads to some easy characterizations.

For the following we need a few results only: The propositions

$$a_{\mathrm{I}}(\vartheta_+) \wedge a_{\mathrm{II}}(\vartheta_-), \qquad a_{\mathrm{I}}(\vartheta'_-) \wedge a_{\mathrm{II}}(\vartheta'_+), \qquad a_{\mathrm{I}}(\vartheta_+) \wedge a_{\mathrm{II}}(\vartheta'_+)$$

$$W := \bigwedge_{\vartheta} \{(a_{\mathrm{I}}(\vartheta_+) \wedge a_{\mathrm{II}}(\vartheta_-)) \vee (a_{\mathrm{I}}(\vartheta_-) \wedge a_{\mathrm{II}}(\vartheta_+))\}$$

are atoms (least elements $\not\equiv \Lambda$) of the lattice $\langle E(S), \wedge, \vee, \neg \rangle$. In the

Hilbert space representation, atoms correspond to vectors which describe "pure states" of the system S. The product states $\varphi^{\mathrm{I}}_{+}(\vartheta)\otimes\varphi^{\mathrm{II}}_{-}(\vartheta)$, $\varphi^{\mathrm{I}}_{-}(\vartheta')\otimes\varphi^{\mathrm{II}}_{+}(\vartheta')$, $\varphi^{\mathrm{I}}_{+}(\vartheta)\otimes\varphi^{\mathrm{II}}_{+}(\vartheta')$ determine that system S_{I} is in the states $\varphi^{\mathrm{I}}_{+}(\vartheta)$, $\varphi^{\mathrm{I}}_{-}(\vartheta')$, $\varphi^{\mathrm{I}}_{+}(\vartheta)$, respectively, and system S_{II} is in the states $\varphi^{\mathrm{II}}_{-}(\vartheta)$, $\varphi^{\mathrm{II}}_{+}(\vartheta')$, $\varphi^{\mathrm{II}}_{+}(\vartheta')$, respectively, which correspond to the atoms of the local language $\mathbb{S}(\mathfrak{R}_{\mathrm{I}})$, namely, $a_{\mathrm{I}}(\vartheta_{+})$, $\neg a_{\mathrm{I}}(\vartheta'_{+})$, $a_{\mathrm{I}}(\vartheta_{+})$, respectively, and of the local language $\mathbb{S}(\mathfrak{R}_{\mathrm{II}})$, namely, $\neg a_{\mathrm{II}}(\vartheta_{+})$, $a_{\mathrm{II}}(\vartheta'_{+})$, $a_{\mathrm{II}}(\vartheta'_{+})$, respectively. Therefore the conjunctions $a_{\mathrm{I}}(\vartheta_{+})\wedge a_{\mathrm{II}}(\vartheta_{-})$, $a_{\mathrm{I}}(\vartheta'_{-})\wedge a_{\mathrm{II}}(\vartheta'_{+})$, and $a_{\mathrm{I}}(\vartheta_{+})\wedge a_{\mathrm{II}}(\vartheta'_{+})$ are atoms of the universal language $\mathbb{S}_{U}$. The ${}^{1}S_0$ state $\phi := (1/\sqrt{2})[\varphi^{\mathrm{I}}_{+}(\vartheta)\otimes\varphi^{\mathrm{II}}_{-}(\vartheta)-\varphi^{\mathrm{I}}_{-}(\vartheta)\otimes\varphi^{\mathrm{II}}_{+}(\vartheta)]$ determines a strong negative correlation with respect to each direction **a** with $a_z=\cos\vartheta$. This means that, if the compound system S is prepared in the state ϕ and a measurement of the observable $\sigma_{\mathrm{I}}(\vartheta)$ is performed in $\mathfrak{R}_{\mathrm{I}}$ with value $+1$, then, after the measurement, the system S is in the state $\varphi^{\mathrm{I}}_{+}(\vartheta)\otimes\varphi^{\mathrm{II}}_{-}(\vartheta)$, and this is the case for arbitrary angles ϑ. It is easy to see that the proposition $W:=\bigwedge_{\vartheta}\{(a_{\mathrm{I}}(\vartheta_{+})\wedge a_{\mathrm{II}}(\vartheta_{-}))\vee(a_{\mathrm{I}}(\vartheta_{-})\wedge a_{\mathrm{II}}(\vartheta_{+}))\}$ satisfies the condition that, after a successive proof of the proposition $a_{\mathrm{I}}(\vartheta_{+})$, the proposition $a_{\mathrm{I}}(\vartheta_{+})\wedge a_{\mathrm{II}}(\vartheta_{-})$ is true, and that this is the case for arbitrary angles ϑ. Namely, the fact that, after the preparation of the system S such that W was true, the proposition $a_{\mathrm{I}}(\vartheta_{+})$ is true, is stated by the sequential conjunction $W\sqcap a_{\mathrm{I}}(\vartheta_{+})$. We show now the following:

Theorem 1:

(a) $$W\sqcap a_{\mathrm{I}}(\vartheta_{+})\leqslant[W\sqcap a_{\mathrm{I}}(\vartheta_{+})]\equiv a_{\mathrm{I}}(\vartheta_{+})\wedge a_{\mathrm{II}}(\vartheta_{-})$$

(b) $$W\sqcap a_{\mathrm{I}}(\vartheta_{+})\sqcap a_{\mathrm{II}}(\vartheta'_{+})\leqslant[W\sqcap a_{\mathrm{I}}(\vartheta_{+})\sqcap a_{\mathrm{II}}(\vartheta'_{+})]$$
$$\equiv a_{\mathrm{I}}(\vartheta_{+})\wedge a_{\mathrm{II}}(\vartheta'_{+})$$

Remark. Here we make use of the result (Stachow, 1980, p. 286) that for any sequential conjunction $\mathfrak{A}$ there exists a logically connected proposition $[\mathfrak{A}]$ which is a supremum for $\mathfrak{A}$ in the sense that $\mathfrak{A}\leqslant[\mathfrak{A}]$ and $\mathfrak{A}\leqslant\mathfrak{B}\cap[\mathfrak{A}]\leqslant\mathfrak{B}$ for any sequential conjunction $\mathfrak{B}$. If W is a material proposition we have

$$W\sqcap a_{\mathrm{I}}(\vartheta_{+})\leqslant[W\sqcap a_{\mathrm{I}}(\vartheta_{+})]\equiv(W\vee\neg a_{\mathrm{I}}(\vartheta_{+}))\wedge a_{\mathrm{I}}(\vartheta_{+})$$

and

$$W\sqcap a_{\mathrm{I}}(\vartheta_{+})\sqcap a_{\mathrm{II}}(\vartheta'_{+})\leqslant[W\sqcap a_{\mathrm{I}}(\vartheta_{+})\sqcap a_{\mathrm{II}}(\vartheta'_{+})]$$
$$\equiv[[W\sqcap a_{\mathrm{I}}(\vartheta_{+})]\sqcap a_{\mathrm{II}}(\vartheta'_{+})]$$

Proof. (a) Inserting the explicit form of W we obtain

$$(W \vee \neg a_{\mathrm{I}}(\vartheta_{+})) \wedge a_{\mathrm{I}}(\vartheta_{+}) \leqslant \{(a_{\mathrm{I}}(\vartheta_{+}) \wedge a_{\mathrm{II}}(\vartheta_{-})) \vee (a_{\mathrm{I}}(\vartheta_{-}) \wedge a_{\mathrm{II}}(\vartheta_{+})) \vee \neg a_{\mathrm{I}}(\vartheta_{+})\} \wedge a_{\mathrm{I}}(\vartheta_{+}) \leqslant a_{\mathrm{I}}(\vartheta_{+}) \wedge a_{\mathrm{II}}(\vartheta_{-})$$

Since $(W \vee \neg a_{\mathrm{I}}(\vartheta_{+})) \wedge a_{\mathrm{I}}(\vartheta_{+}) \not\equiv \Lambda$ and $a_{\mathrm{I}}(\vartheta_{+}) \wedge a_{\mathrm{II}}(\vartheta_{-})$ is an atom, we finally have

$$[W \sqcap a_{\mathrm{I}}(\vartheta_{+})] \equiv (W \vee \neg a_{\mathrm{I}}(\vartheta_{+})) \wedge a_{\mathrm{I}}(\vartheta_{+}) \equiv a_{\mathrm{I}}(\vartheta_{+}) \wedge a_{\mathrm{II}}(\vartheta_{-}) \qquad \blacksquare$$

Similarly we have

$$[W \sqcap a_{\mathrm{II}}(\vartheta'_{+})] \equiv (W \vee \neg a_{\mathrm{II}}(\vartheta'_{+})) \wedge a_{\mathrm{II}}(\vartheta'_{+}) \equiv a_{\mathrm{I}}(\vartheta'_{-}) \wedge a_{\mathrm{II}}(\vartheta'_{+})$$

(b) $$[[W \sqcap a_{\mathrm{I}}(\vartheta_{+})] \sqcap a_{\mathrm{II}}(\vartheta'_{+})] \equiv ((a_{\mathrm{I}}(\vartheta_{+}) \wedge a_{\mathrm{II}}(\vartheta_{-})) \vee \neg a_{\mathrm{II}}(\vartheta'_{+})) \wedge a_{\mathrm{II}}(\vartheta'_{+})$$

$$\leqslant (a_{\mathrm{I}}(\vartheta_{+}) \vee \neg a_{\mathrm{II}}(\vartheta'_{+})) \wedge (a_{\mathrm{II}}(\vartheta_{-}) \vee \neg a_{\mathrm{II}}(\vartheta'_{+})) \wedge a_{\mathrm{II}}(\vartheta'_{+})$$

$$\equiv (a_{\mathrm{I}}(\vartheta_{+}) \vee \neg a_{\mathrm{II}}(\vartheta'_{+})) \wedge a_{\mathrm{II}}(\vartheta'_{+}), \qquad \text{since } a_{\mathrm{II}}(\vartheta_{-}) \vee \neg a_{\mathrm{II}}(\vartheta'_{+}) \equiv V$$

$$\equiv ((a_{\mathrm{I}}(\vartheta_{+}) \wedge a_{\mathrm{II}}(\vartheta'_{+})) \vee ((\neg a_{\mathrm{II}}(\vartheta'_{+}) \wedge a_{\mathrm{II}}(\vartheta'_{+})) \equiv a_{\mathrm{I}}(\vartheta_{+}) \wedge a_{\mathrm{II}}(\vartheta'_{+})$$

Since $a_{\mathrm{I}}(\vartheta_{+}) \wedge a_{\mathrm{II}}(\vartheta'_{+})$ is an atom, we obtain

$$[[W \sqcap a_{\mathrm{I}}(\vartheta_{+})] \sqcap a_{\mathrm{II}}(\vartheta'_{+})] \equiv a_{\mathrm{I}}(\vartheta_{+}) \wedge a_{\mathrm{II}}(\vartheta'_{+}) \qquad \blacksquare$$

If, after the preparation W of the system $S = S_{\mathrm{I}} + S_{\mathrm{II}}$, the proposition $a_{\mathrm{II}}(\vartheta'_{+})$ has been tested but the proof result is not known to the observer, his information is confined to the truth of the proposition $W \sqcap (a_{\mathrm{II}}(\vartheta'_{+}) \cup a_{\mathrm{II}}(\vartheta'_{-}))$. The supremum of this sequential conjunction, which represents the least logically connected proposition (containing the greatest information) which can be predicted in a situation where the sequential conjunction is known, is given by

$$W \sqcap (a_{\mathrm{II}}(\vartheta'_{+}) \cup a_{\mathrm{II}}(\vartheta'_{-})) \leqslant [W \sqcap (a_{\mathrm{II}}(\vartheta'_{+}) \cup a_{\mathrm{II}}(\vartheta'_{-}))] \equiv [W \sqcap a_{\mathrm{II}}(\vartheta'_{+})] \cup [W \sqcap a_{\mathrm{II}}(\vartheta'_{-})]$$

If probability propositions are added to the formal language, the information after the test of $a_{\mathrm{II}}(\vartheta'_+)$ without knowing the result may be formulated by the *Gemenge*

$$\Gamma(W; a_{\mathrm{II}}(\vartheta'_+)) := \{p_{\langle W\rangle}(a_{\mathrm{II}}(\vartheta'_+)), p_{\langle W\rangle}(a_{\mathrm{II}}(\vartheta'_-)); a_{\mathrm{II}}(\vartheta'_+), a_{\mathrm{II}}(\vartheta'_-)\}$$

where the probability expressions represent the conditional probabilities for the proofs of $a_{\mathrm{II}}(\vartheta'_+)$ and $a_{\mathrm{II}}(\vartheta'_-)$ after the system has been prepared in W. For the introduction of probability propositions into the formal quantum language compare Stachow (1979, 1981c). Hence we refer to $W \sqcap (a_{\mathrm{II}}(\vartheta'_+) \sqcup a_{\mathrm{II}}(\vartheta'_-))$ as the *sequential Gemenge proposition* and to $[W \sqcap (a_{\mathrm{II}}(\vartheta'_+) \sqcup a_{\mathrm{II}}(\vartheta'_-))]$ as the *material Gemenge proposition.*

Another interesting question concerns the least propositions $W(S_{\mathrm{I}})$ and $W(S_{\mathrm{II}})$ with respect to the systems S_{I} and S_{II} which can be predicted if it is known that the compound system $S = S_{\mathrm{I}} + S_{\mathrm{II}}$ is prepared in W.

Theorem 2. $W(S_{\mathrm{I}}) \equiv V_{\mathrm{I}}$, $W(S_{\mathrm{II}}) \equiv V_{\mathrm{II}}$.

Proof. The least proposition $W(S_{\mathrm{I}})$ is an element of the lattice $\langle E(S_{\mathrm{I}}), \wedge, \vee, \neg\rangle$. Let us assume that $W(S_{\mathrm{I}}) \equiv a_{\mathrm{I}}(\vartheta_+)$ for some angle ϑ. Then we have $W \leqslant a_{\mathrm{I}}(\vartheta_+)$ also and therefore $W \equiv W \wedge a_{\mathrm{I}}(\vartheta_+)$. But

$$\begin{aligned} W \wedge a_{\mathrm{I}}(\vartheta_+) &\leqslant \{(a_{\mathrm{I}}(\vartheta_+) \wedge a_{\mathrm{II}}(\vartheta_-)) \vee (a_{\mathrm{I}}(\vartheta_-) \wedge a_{\mathrm{II}}(\vartheta_+))\} \wedge a_{\mathrm{I}}(\vartheta_+) \\ &\equiv (a_{\mathrm{I}}(\vartheta_+) \wedge a_{\mathrm{II}}(\vartheta_-) \wedge a_{\mathrm{I}}(\vartheta_+)) \vee (a_{\mathrm{I}}(\vartheta_-) \wedge a_{\mathrm{II}}(\vartheta_+) \wedge a_{\mathrm{I}}(\vartheta_+)) \\ &\equiv a_{\mathrm{I}}(\vartheta_+) \wedge a_{\mathrm{II}}(\vartheta_-) \end{aligned}$$

Since W and $a_{\mathrm{I}}(\vartheta_+) \wedge a_{\mathrm{II}}(\vartheta_-)$ are different atoms, it follows that $W \wedge a_{\mathrm{I}}(\vartheta_+) \equiv \Lambda$, a contradiction to $W \equiv W \wedge a_{\mathrm{I}}(\vartheta_+)$. Therefore, it is only possible that $W(S_{\mathrm{I}}) \equiv V_{\mathrm{I}}$, and $W(S_{\mathrm{II}}) \equiv V_{\mathrm{II}}$ analogously. ∎

The propositions $W(S_{\mathrm{I}})$ and $W(S_{\mathrm{II}})$ are called the *reductions* of the proposition $W =: W(S) = W(S_{\mathrm{I}} + S_{\mathrm{II}})$ with respect to the systems S_{I} and S_{II}.

3.2. Relativistic Considerations. A formal description of the EPR experiment, which is convenient for an adequate treatment of the locality principle, the nonlocal correlations, and the causality problem, must take into account quantum physical as well as relativistic aspects. This means that the formal quantum language $\mathfrak{S}_U$ must be further specified in order to take into account also the principles of special relativity. A relativistic quantum language of this kind was recently developed systematically (Mittelstaedt, 1982), to which we will refer here.

The preparation W of the system S as well as other measurable propositions $A, B, \ldots$ are assumed here to be time independent in the sense of a Heisenberg representation. In the framework of the relativistic quantum language this means for the present that these propositions are related to the entire Minkowskian space-time $\mathfrak{M}$. On the other hand, the measuring process for a given proposition A is performed in a finite region $\mathfrak{R} \subseteq \mathfrak{M}$ of space-time and hence the validity of this proposition cannot be extended to the entire space-time $\mathfrak{M}$ but must be restricted to a certain validity region $M = M(A) \subseteq \mathfrak{M}$.

According to the investigations mentioned, the validity region of a proposition A, which is measured at the event point $x^* \in \mathfrak{M}$ is defined as follows.[1] If W is the initial preparation of the system S, the history of S due to the A measurement is described by $W \sqcap A$ and the state after the measurement by $[W \sqcap A] \equiv (W \vee \neg A) \wedge A$. The validity region $M_0 = M_0(W)$ of the initial proposition W is then given by the causal past $J^{(-)}(x^*)$ of the event x^* (Hawking et al., 1973) and the validity region $M_1 = M_1([W \sqcap A])$ for the state after the measurement by the complement $\bar{J}^{(-)}(x^*)$ of the set M_0 with respect to $\mathfrak{M}$. Clearly the region M_1 includes also events z^*, which have spacelike distance to the measuring event x^*.

If once the validity regions for a single measuring process are defined in this way, one can consider the more complicated measuring program, which consists of two measuring processes for propositions A and B at event points x^* and y^*, respectively. If the events x^* and y^* have timelike distance with $x_0^* < y_0^*$, say, then we have two subsequent measurements described by $W \sqcap A$ and $(W \sqcap A) \sqcap B$, and consequently three validity regions $M_0 = J^{(-)}(x^*)$, $M_1 = J^{(-)}(y^*) \cap \bar{J}^{(-)}(x^*)$, and $M_2 = \bar{J}^{(-)}(y^*)$ which correspond to W, $[W \sqcap A]$, and $[W \sqcap A \sqcap B]$, respectively. If the events x^* and y^* of the measurement have spacelike distance, i.e., $x^* \sim y^*$, the history of S and the corresponding validity regions are more complicated. Here we have to distinguish four validity regions $M_0 = J^{(-)}(x^*) \cap J^{(-)}(y^*)$, $M_1 = J^{(-)}(x^*) \cap \bar{J}^{(-)}(y^*)$, $M_2 = \bar{J}^{(-)}(x^*) \cap J^{(-)}(y^*)$, and $M_3 = \bar{J}^{(-)}(x^*) \cap \bar{J}^{(-)}(y^*)$ which correspond to the propositions W, $[W \sqcap A]$, $[W \sqcap B]$, and $[W \sqcap A \sqcap B]$. A division of the Minkowskian space-time $\mathfrak{M}$ into distinct regions M_i which are determined by measuring processes will be called an M chart of $\mathfrak{M}$.

If we apply these considerations to the EPR experiment, we obtain the following description of the history of the system $S = S_{\mathrm{I}} + S_{\mathrm{II}}$. The measuring program of this experiment consists of two parts, a measurement of the proposition $a_{\mathrm{I}}(\vartheta_+)$ at the event point x^* and a measurement of $a_{\mathrm{II}}(\vartheta'_+)$ at

[1]For simplicity's sake we will assume here that the finite region $\mathfrak{R} \subseteq \mathfrak{M}$ in which the measuring is performed can be replaced by a single event point $x^* \in \mathfrak{R}$. For the present investigation this idealization is not problematic.

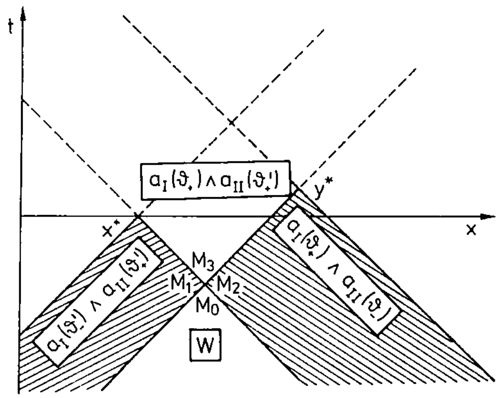


Fig. 4. The history of S in Minkowskian space-time.

the event y^*. The points x^* and y^* have spacelike distance, $x^* \sim y^*$, and the coordinate system of the observer is chosen such that $x_0^* < y_0^*$. If we assume that the actual results of the measurements at x^* and y^* are $a_{\mathrm{I}}(\vartheta_+)$ and $a_{\mathrm{II}}(\vartheta'_+)$, the history of S is completely determined and can be represented by the respective M chart in $\mathfrak{M}$ (Figure 4). The initial preparation W is relevant for the region $M_0 = J^{(-)}(x^*) \cap J^{(-)}(y^*)$. In $M_1 = J^{(-)}(x^*) \cap \bar{J}^{(-)}(y^*)$ we have to take into account the result of the measurement at y^* but not that at x^*. For this reason and on account of the correlation between S_{I} and S_{II}, we have in M_1 the state $[W \sqcap a_{\mathrm{II}}(\vartheta'_+)] \equiv a_{\mathrm{I}}(\vartheta'_-) \wedge a_{\mathrm{II}}(\vartheta'_+)$. Analogously we obtain for $M_2 = \bar{J}^{(-)}(x^*) \cap J^{(-)}(y^*)$ the state $[W \sqcap a_{\mathrm{I}}(\vartheta_+)] \equiv a_{\mathrm{I}}(\vartheta_+) \wedge a_{\mathrm{II}}(\vartheta_-)$. In the last region $M_3 = \bar{J}^{(-)}(x^*) \cap \bar{J}^{(-)}(y^*)$ we have the final result of the EPR experiment, namely, the state $[W \sqcap a_{\mathrm{I}}(\vartheta_+) \sqcap a_{\mathrm{II}}(\vartheta'_+)] \equiv a_{\mathrm{I}}(\vartheta_+) \wedge a_{\mathrm{II}}(\vartheta'_+)$ (cf. Theorem 1).

However, the following point of view must be taken into account. The history of the system just described is known only to an "ultimate" observer, who is completely informed of the measuring program and of the outcomes of the $a_{\mathrm{I}}(\vartheta_+)$ trial in x^* and the $a_{\mathrm{II}}(\vartheta'_+)$ trial in y^*. According to special relativity and Einstein causality, classical signals (light, sound, etc.) can be transmitted from x^*, say, only to points $z^* \in J^{(+)}(x^*)$, where $J^{(+)}(x^*)$ is the causal future of x^*. This means that the described history of S is relevant only for an observer B_U, the space-time point $\xi(B_U)$ of which is contained in the domain $J^{(+)}(x^*) \cap J^{(+)}(y^*)$.

All other observers are only partly informed about the outcomes of the $a_{\mathrm{I}}(\vartheta_+)$—and $a_{\mathrm{II}}(\vartheta'_+)$—trials at x^* and y^*, respectively, and will thus give an incomplete "partial" description of the history of S. However, we assume that all observers are completely informed about the measuring

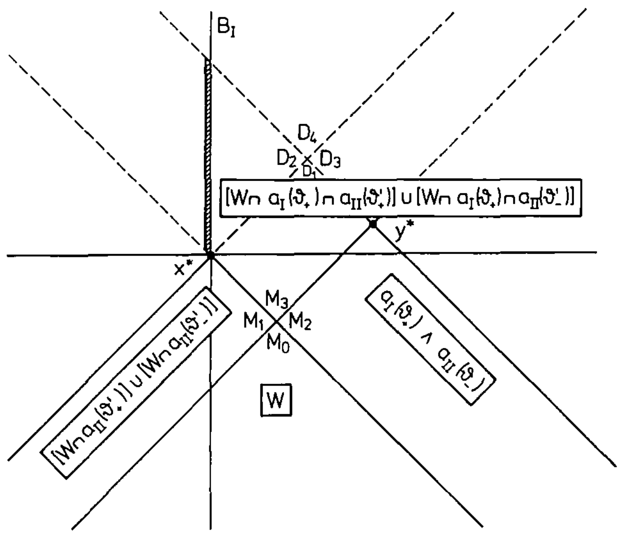


Fig. 5. The partial description of the history of S by the observer B_{I} with position $\xi \in D_2$.

program. Generally we have to distinguish four distinct domains D_1, D_2, D_3, D_4 of $\mathfrak{M}$ which correspond to different possibilities of information and which are given by (Figure 5).

$$D_1 = \bar{J}^{(+)}(x^*) \cap \bar{J}^{(+)}(y^*), \qquad D_2 = J^{(+)}(x^*) \cap \bar{J}^{(+)}(y^*)$$

$$D_3 = \bar{J}^{(+)}(x^*) \cap J^{(+)}(y^*), \qquad D_4 = J^{(+)}(x^*) \cap J^{(+)}(y^*)$$

For the partial description of the history of S by an arbitrary observer B, it is important which information is available for him, i.e., in which of these domains D_i his space-time point $\xi(B)$ is located.

These considerations are important in particular for the observers B_{I} and B_{II}, who actually perform the measurements in x^* and y^*. Let us assume that the position $\xi(B_{\text{I}})$ of B_{I} is in D_2 (Figure 5). The history of S described by B_{I} then begins again with the preparation W in M_0. In region M_1 the outcome of the $a_{\text{II}}(\vartheta'_+)$ trial in y^* would be relevant, but this result is not available for B_{I}. According to the measuring program B_{I} knows only that in y^* the $a_{\text{II}}(\vartheta'_+)$ trial was performed at all. Hence he describes the development of S in M_1 by the sequential "Gemenge" proposition $W \sqcap (a_{\text{II}}(\vartheta'_+) \cup a_{\text{II}}(\vartheta'_-))$ and the state of S in this region by the logical "Gemenge" proposition $[W \sqcap a_{\text{II}}(\vartheta'_+)] \cup [W \sqcap a_{\text{II}}(\vartheta'_-)]$. In M_2 the outcome of the $a_{\text{I}}(\vartheta_+)$ trial at x^* is relevant and here the observer B_{I} knows the

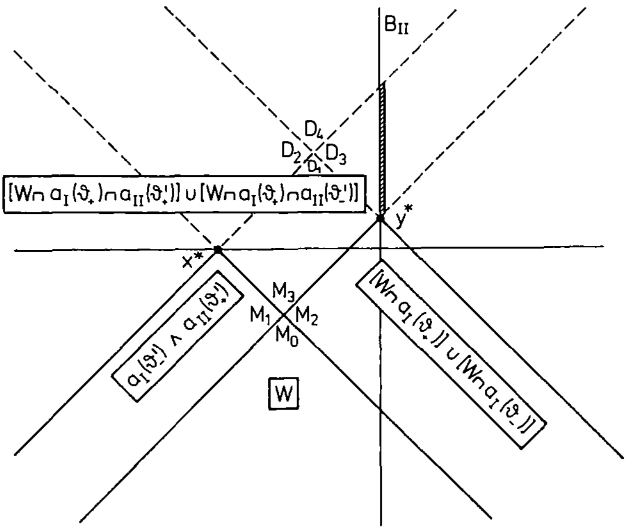


Fig. 6. The partial description of the history of S by the observer B_{II} with position $\xi \in D_3$.

actual result. Hence he describes S by the same state $a_{\text{I}}(\vartheta_+) \wedge a_{\text{II}}(\vartheta_-)$ as in the complete description of the history of S. Finally, in M_3 the observer B_{I} has to take into account the outcome of the $a_{\text{I}}(\vartheta_+)$ trial at x^* and the fact that the $a_{\text{II}}(\vartheta'_+)$ measurement at y^* was performed at all. Hence he describes the state of S in M_3 by the disjunction of the two states $[W \sqcap a_{\text{I}}(\vartheta_+) \sqcap a_{\text{II}}(\vartheta'_+)]$ and $[W \sqcap a_{\text{I}}(\vartheta_+) \sqcap a_{\text{II}}(\vartheta'_-)]$ which correspond to the two alternatives of the sequential "Gemenge" proposition $W \sqcap (a_{\text{II}}(\vartheta'_+) \sqcup a_{\text{II}}(\vartheta'_-))$ in M_1 (Figure 5). Completely analogous considerations hold for the observer B_{II} with a position $\xi(B_{\text{II}}) \in D_3$. (Figure 6).

4. THE RESOLUTION OF THE EPR PARADOX

4.1. The Principles *R*, *L*, and *Q*. In the framework of the universal language $\mathfrak{S}_U$, which was applied to the EPR experiment in Section 3, we reconsider now the contradiction between R, L, and Q demonstrated in Section 2. The principle Q, which states the validity of quantum physics, is assumed to be true throughout this paper. It is empirically well established and basic for the universal language $\mathfrak{S}_U$. The "reality principle" R was accepted in Section 2 as a necessary precondition of a language about a physical system and its properties. Hence within the framework of $\mathfrak{S}_U$ this principle should turn out to be already incorporated in this language. The premise R_1 states that a proposition A, say, can be tested at the system S

with preparation W without thereby influencing the system in any way. Since in $\mathfrak{S}_U$ the successful test of A is described by $W \sqcap A$, the premise R_1 reads $W \equiv W \sqcap A$. The conclusion R_2 states that the system S possesses a property $\mathcal{P}_A$, which corresponds to A, i.e., $S(W) \vdash A$ or $W \leqslant A$. The "reality principle" $R = R_1 \cap R_2$ can thus be expressed by

$$(R) \qquad W \equiv W \sqcap A \cap S(W) \vdash A$$

It is obvious that R is not a new principle which can be added to the language $\mathfrak{S}_U$, since it is already incorporated into $\mathfrak{S}_U$. In this language it appears as a criterion for the truth of A in the system S with preparation W, i.e., for $S(W) \vdash A$. If $S(W) \vdash A$ holds, we also say that the property $\mathcal{P}_A$ which corresponds to A is "real" in S.

If once the *reality* of a property $\mathcal{P}_A$ at system $S(W)$ is explained by $W \leqslant A$, the *objectivity* of $\mathcal{P}_A$ at $S(W)$ can be defined by: $W \leqslant A$ or $W \leqslant \neg A$ (Mittelstaedt, 1979; Burghardt, 1980). The objectivity of A in $S(W)$ means that it is objectively decided whether A or $\neg A$ holds in $S(W)$, but subjectively unknown. Hence the objectivity concept incorporates both the objective decidedness and the subjective ignorance of the observer. It is well known that in general a proposition A is not objective with respect to W. However, by a measuring process for A the initial state W is in a first step transformed into a Gemenge $\Gamma = \Gamma(W; A)$ of two alternatives. The proposition A is thus objective with respect to Γ. In a second step the actual measuring result can then be obtained simply by reading the outcome of the measurement.

The "locality principle" $L = L_1 \to L_2$ was already questioned in Section 2 and will in fact turn out to be the origin of the EPR contradiction. The premise L_1 states that two systems S_{I} and S_{II} cannot interact. The preparation W of the EPR system corresponds to the ${}^1S_0$ state of two subsystems which do not interact. Moreover, since the event points x^* and y^* of the measuring processes are assumed to have spacelike distance, the measuring process at x^* cannot lead to an interaction between S_{I} and the system S_{II} at y^*. Hence the statement of the premise L_1 is realized by the EPR experiment and thus the implication $L = L_1 \to L_2$ can be applied to this situation.

The conclusion L_2 states that the measuring process at system S_{I}, say, cannot influence the other system S_{II} in any way. However, this statement does not correspond to the EPR situation. According to the M chart of $S_{\text{I}} + S_{\text{II}}$ the change in the state description due to the measuring process at x^* is relevant for all events $z^* \in \bar{J}^{(-)}(x^*)$, in particular for the event y^* of the S_{II} measurement. Hence by an $a_{\text{I}}(\vartheta_+)$ measurement at S_{I} in x^* the state $W(S_{\text{II}}) = V_{\text{II}}$ of system S_{II} is transformed into the Gemenge $\Gamma_{\text{II}}(\vartheta) = \Gamma(W(S_{\text{II}}); a_{\text{II}}(\vartheta_+))$, i.e., the observable $a_{\text{II}}(\vartheta_+)$ is objectified in S_{II}, even if the two subsystems have spacelike distance.

We thus find that according to the locality principle $L = L_1 \cap L_2$ the measuring process of $a_I(\vartheta_+)$ at S_I in x^* should not have any influence on the system S_{II} in y^*, whereas according to the relativistic quantum language due to this measuring process the observable $a_{II}(\vartheta_+)$ is objectified in S_{II} and that irrespective of the distance between S_I and S_{II}. Consequently the locality principle L does not hold generally in relativistic quantum physics, and in particular not in the case of the EPR experiment. Hence it is obvious that the contradiction between Q, R, and L mentioned above can be traced back to the invalidity of the locality principle L.

4.2. The Relaxation of the Locality Principle. Since the locality principle does not hold generally in relativistic quantum physics, we are now going to formulate the relaxation of the locality principle L mentioned in Section 2, the weak locality principle $\tilde{L}$, which fulfills the following three requirements:

(i) $L \cap \tilde{L}$
(ii) $\neg(\tilde{L} \cap \neg K)$
(iii) $\neg(\tilde{L} \& R \& Q \cap \Lambda)$

A weak locality principle $\tilde{L}$, which is in accordance with these postulates, reads $\tilde{L} = L_1 \cap \tilde{L}_2$ and

$\tilde{L}_2 :=$ The measurement of an observable A_I at S_I can at least have the effect that an observable A_{II} is objectified in S_{II}.

It is obvious that $\tilde{L}_2$ is a relaxation of L_2, which is sufficiently weak in order to allow for the objectivation of observables in spacelike distances in accordance with the state description by means of M charts. Hence we have $L_2 \cap \tilde{L}_2$ and $L \cap \tilde{L}$ such that the requirement (i) is fulfilled.

According to the second postulate the relaxation $\tilde{L}$ must be sufficiently strong in order to prevent violations of Einstein causality by means of superluminal signals. In the relativistic quantum language $\mathfrak{S}_U$, the descriptions of the quantum physical history of a system S, which are given by different observers B, B', must not contradict each other. This requirement leads to a consistency postulate for $\mathfrak{S}_U$ (C postulate), which states that propositions A and B, which are provable at events x^* and y^* with spacelike distance, are commensurable. Together with a theorem (L theorem) concerning the measuring process one thus arrives at the important result (LC theorem) that the probability $p_{\langle W \rangle}(B)$ of B in y^* with respect to W is the same as the probability $p_{\langle \Gamma(W;A) \rangle}(B)$ of B with respect to the Gemenge $\Gamma(W; A)$ which is generated from W by a test of the proposition A in x^*, provided x^* and y^* have spacelike distance.

weaker locality principle $\tilde{L}$, which can be founded within the framework of the relativistic quantum language $\mathcal{S}_U$. In this way the EPR paradox is resolved.

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